Webinar: Höhere Mathematik 1
Thema: Nullstellen, Grenzwerte, Stetigkeit

Notiztitel 09.08.2016

a)

1) $f(x) = 2x^2 + 4x - 15$ $\qquad a_n = 2$

$$\boxed{f(x) = x^2 + 2x - 7{,}5}$$

p/q-Formel:

$$x_{1,2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$$

$p = 2$

$q = -7{,}5$

$$x_{1,2} = -\frac{2}{2} \pm \sqrt{\left(\frac{2}{2}\right)^2 + 7{,}5}$$

$$x_{1,2} = -1 \pm \sqrt{8{,}5}$$

$x_1 = 1{,}92$ $\quad x_2 = -3{,}92$

2) $f(x) = 4x^4 + 10x^2 - 20 \quad /:4$

$$\boxed{f(x) = x^4 + 2{,}5x^2 - 5}$$

$\boxed{x^2 = z}$ Substitution

$$f(x) = z^2 + 2{,}5z - 5$$

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$$z_{1,2} = -\frac{2,5}{2} \pm \sqrt{\left(\frac{2,5}{2}\right)^2 + 5}$$

$$z_{1,2} = -1,25 \pm \sqrt{6,5625}$$

$$z_1 = 1,312$$

$$\boxed{z_2 = -3,812}$$ Komplexe Nullstelle

$$\boxed{z = x^2}$$

$$x^2 = 1,312$$

$$\boxed{\begin{aligned} x_1 &= \sqrt{1,312} = 1,15 \\ x_2 &= -\sqrt{1,312} = -1,15 \end{aligned}}$$

3) $f(x) = x^3 - x^2 - 8x + 12$

- Teiler von $a_0 = 12$

$1, 2, 3, 4, 6, 12$

$$\boxed{x_1 = 2}$$

$$(x^3 - x^2 - 8x + 12) : (x - 2) = x^2 + x - 6$$

$$\begin{array}{l}
-(x^3 - 2x^2) \\
\hline
0 + x^2 - 8x \\
\quad -(x^2 - 2x) \\
\hline
\quad\quad 0 - 6x + 12 \\
\quad\quad -(-6x + 12) \\
\hline
\quad\quad\quad\quad 0
\end{array}$$

$$x_{2,3} = -\frac{1}{2} \pm \sqrt{\left(\frac{1}{2}\right)^2 + 6}$$

$x_2 = 2$ $\quad$ $x_3 = -3$ $\quad$ $x_1 = 2$

doppelte Nullstelle

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4) $f(x) = \frac{2x^2 + 2x - 12}{6x^2 - 12x}$ $\frac{z(x)}{n(x)}$

$z(x) = 2x^2 + 2x - 12 \quad | :2$

$z(x) = x^2 + x - 6$

$x_{1,2} = -\frac{1}{2} \pm \sqrt{\left(\frac{1}{2}\right)^2 + 6}$

$\boxed{x_1 = 2}$ $\boxed{x_2 = -3}$ Nullstelle der Funktion

$\boxed{n(x=2) = 0}$ Definitionslücke

$n(x=-3) \neq 0$ $\rightarrow$ Nullstelle der Funktion

Polstelle: $n(x) = 0$, $z(x) \neq 0$

mögliche hebbare DL: $n(x) = 0$, $z(x) = 0$

$\boxed{\begin{array}{l} x_1 = 2 \\ x_3 = 2 \end{array}}$ $\quad x_2 = -3 \quad$ Zähler

$\quad x_4 = 0 \quad$ Nenner

$$f(x) = \frac{(x-2)\cdot(x+3)}{(x-2)\cdot(x-0)} = \frac{\cancel{(x-2)}\cdot(x+3)}{\cancel{(x-2)}\cdot x}$$

$$f(x) = \frac{x+3}{x}$$

$$f(x) = \frac{x+3}{x}$$

$n(x=1) = 2 \neq 0$ keine DL

$\boxed{\rightarrow \text{hebbare DL}}$

$$f(x=1) = \frac{5}{2} = 2{,}5$$

$$f(x) = \begin{cases} 2{,}5 & \text{für } x = 2 \\ f(x) & \text{sonst} \end{cases}$$

1) $f(x) = 2x^2 + 4x - 15$

$$f(x) = 2x^2\left(1 + \frac{4x}{2x^2} - \frac{15}{2x^2}\right)$$

$$f(x) = \boxed{2x^2}\left(1 + \underbrace{\frac{2}{x}}_{\to 0} - \underbrace{\frac{15}{2x^2}}_{\to 0}\right)$$

$$\lim_{x \to \infty} 2x^2 = +\infty \qquad \lim_{x \to -\infty} 2x^2 = +\infty$$

2) $f(x) = 4x^4 + 10x^2 - 20$

$$f(x) = 4x^4 \cdot \left(1 + \dots\right)$$

gegen 1

$$\lim_{x \to \infty} 4x^4 = +\infty$$

$$\lim_{x \to -\infty} 4x^4 = +\infty$$

3) $f(x) = x^3 - x^2 - 8x + 12$

$f(x) = x^3 \cdot (1 - \ldots\ldots)$

gegen 1

$$\lim_{x \to \infty} x^3 = +\infty$$

$$\lim_{x \to -\infty} x^3 = -\infty$$

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4) $f(x) = \dfrac{2x^2 + 2x - 12}{6x^2 - 12x}$

$$f(x) = \frac{2x^2 (1 + \; \ldots \;)}{6x^2 \cdot (1 - \; \ldots \;)}$$

$$\lim_{x \to \infty} \frac{2\cancel{x^2}}{6\cancel{x^2}} = \frac{2}{6} = \frac{1}{3}$$

$$\lim_{x \to -\infty} \frac{2x^2}{6x^2} = \frac{2}{6} = \frac{1}{3}$$

c)

$$f(x) = x^3 - x^2 - 8x + 12 \qquad x = x_0$$

$$\boxed{f(x = x_0) = x_0^3 - x_0^2 - 8x_0 + 12}$$

$$\lim_{h \to 0} f(x_0 + h) = \lim_{h \to 0} f(x_0 - h) = f(x_0)$$

$$\lim_{h \to 0} f(x_0+h) = \lim_{h \to 0} (x_0+h)^3 - (x_0+h)^2 - 8 \cdot (x_0+h) + 12$$

$$(x_0+h)^2 \cdot (x_0+h)$$

$$\lim_{h \to 0} \left(x_0^3 - x_0^2 - 8x_0 + 12 \right)$$

$$\lim f(x_0-h) = \lim_{h \to 0} (x_0-h)^3 - (x_0-h)^2 - 8 \cdot (x_0-h) + 12$$

$$\lim f(x_0-h) \; \left(x_0^3 - x_0^2 - 8x_0 + 12 \right)$$

$\longrightarrow$ Stetig!!

4) $$\frac{2x^2+2x-12}{6x^2-12x}$$

$x_1 = 2 \quad x_2 = 0 \rightarrow$ Nennernullstellen: unstetig

$(-\infty, 2) \quad (2, 0) \quad (0, \infty)$ stetig in diesem Definitionsbereich